Performance of OFDM Based Amplify-and-Forward Relay in Nakagami Fading Channel

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Abstract

In this paper, the performance of single relay cooperative communication applying orthogonal frequency division multiplexing (OFDM) signals has been investigated in Nakagami fading channel. The signal to interference noise ratio and average bit error rate both without and with diversity has been developed in the impact of frequency offset and phase noise. Numerical results showed a better system performance when higher order Nakagami fading parameter *m* was used with lower frequency offset.

Keywords

Amplify-and-forward; Frequency Offset; Nakagami Fading; Orthogonal Frequency Division Multiplexing and Phase Noise

Introduction

Cooperative communication, one of the fastest growing research areas at this time, is a suitable technology for efficient spectrum use in fourth generation. There are three major objectives of the 4G technologies to fulfill the requirements-continuous connectivity, data rate of 100 Mbps at user terminal and other services like intelligent transportation systems (ITS) deployment [Sinha et al., 2010]. Orthogonal frequency division multiplexing (OFDM) based cooperative wireless communications is a novel technique that allows users to intertransmit messages to the intended destination. The proposed schemes can increase the system reliability by achieving spatial cooperative diversity and also increase their effective quality of service via cooperation. Cooperation allows single antenna nodes in a multiuser environment to share their antennas with other nodes in a distributed manner so that a node can realize a virtual multiantenna [Sreedhar et al., 2008].

Cooperative diversity replicates the performance benefits of MIMO systems that are achieved by the transmission through additional relays [Sohaib et al., 2012]. This cooperative diversity leads to increased exponential decay rate in the error probability with increasing signal to noise ratio [Li et al., 2010]. Various methods have been proposed to achieve the goal such as amplify and forward (AF), decode and forward (DF), coded cooperation (CC) and Hybrid DF-AF [Hong et al., 2010]. The benefit of AF relay protocol is simple and low cost implementation. But the noise is also amplified at the relay. DF relays decode the information before transmission and if relay cannot remove the original message, it is meaningless to transmit the signal at the destination. AF relays forward the signal without hard decoding [Michalopoulos et al., 2012].

OFDM technology has been widely used in modern wireless communication due to its high spectral efficiency, high tolerance to multi-path interference and low complex receivers [Fazel et al., 2008]. It holds a major advantage that a frequency selective channel is converted into multiple frequency flat channels [Larsson et al., 2003]. OFDM has two main disadvantages: inter carrier interference (ICI) and peak to average power ratio (PAPR). ICI occurs due to frequency offset and phase noise among source, relay and destination.

Fading arise as a result of multi-path propagation for the transmitted signal in wireless communication systems. Although a lot of works rely on a Rayleigh fading and Rician fading, Nakagami-*m* fading covers a broader variety of fading scenarios by changing the value of the Nakagami parameter *m*. For radio transmission over a Nakagami fading channel, OFDM based cooperative communication technique need less transmission energy for the same signal to interference ratio (SNIR) requirement and mimic transmit antenna diversity to mitigate fading in the wireless communication.

In this paper, we derives with and without diversity

expressions of SNIR and average bit error rate (BER) for the AF relaying with a single relay in Nakagami fading channel. In addition, the effect of frequency offset and phase noise is included to our proposed scheme and evaluated by numerical simulation.

System Model

A basic OFDM based amplify-and-forward (AF) relay is shown in figure 1. In this scheme, the source data are modulated into OFDM modulation. Then it simultaneously transmits information symbols. The outputs are subsequently sent into two ways in phase I and phase II.

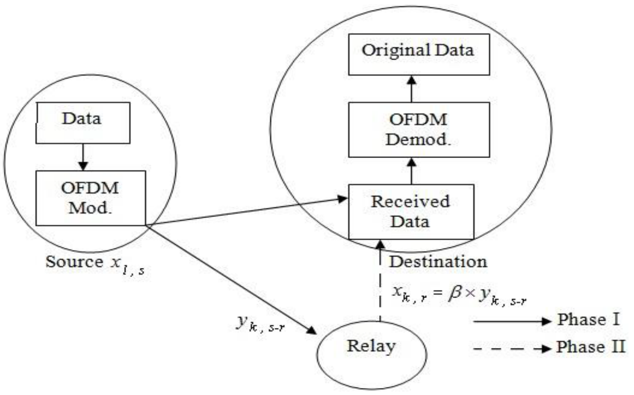

FIG. 1 DIAGRAM OF OFDM BASED AMPLIFY-AND-FORWARD RELAY SCHEME

In phase I, the data transfer source to destination and source to relay directly. On the other hand, in phase II data are transfer from relay to destination. One user acts as the source while the other user serves as the relay and the two users interchange their roles as source and relay at different instants in time. In both cases, the relay enhances communication between the source and destination. Each cooperative relay has to receive the signal and to amplify the received signal. Then each relay has to forward its amplified signal through a wireless channel to the destination. In the destination, the received signals are subsequently passed through OFDM demodulator. Finally it is fed into original signal after OFDM demodulation.

Theoretical Analysis of Amplify and Forward Relay

The baseband signal is passed through the OFDM modulator. The OFDM signal to be transmitted at the transmitter can be expressed as [Li et al., 2008],

$$x(n) = \sum_{k=0}^{N-1} x_{k,s} . e^{j(\frac{2\pi}{N})kn} \quad \text{for } 0 \le \text{n} \le \text{N-1} \tag{1}$$

Where, $\text{j} = \sqrt{-1}$, after passing through the channel, the received signal is affected by phase noise and frequency offset and can be expressed as [Li et al., 2008],

$$r(n) = [x(n) \otimes h(n) + w(n)] e^{j[2\pi\Delta f t + \phi(n)]} \tag{2}$$

Where, Δf and $\phi(n)$ are frequency offset and phase noise. $x(n)$, $h(n)$, $w(n)$, $r(n)$ are transmitted signal, channel impulse response, additive white Gaussian noise (AWGN) and received signal, respectively.

In AF relay scheme, the system works into two phases. In Phase I, the source transmits to the same signal both relay and destination. The source-relay and the source-destination received signals are given by

$$\begin{aligned} y_{k,s-r} &= \sum_{l=0}^{N-1} x_{l,s} H_{l,s-r} C_{l-k} + w_r \\ &= x_{k,s} H_{k,s-r} C_0 + \sum_{l=0, l \ne k}^{N-1} x_{l,s} H_{k,s-r} C_{l-k} + w_r \\ &= x_{k,s} H_{k,s-r} C_0 + I_{ICI(s-r)} + w_r \end{aligned} \tag{3}$$

$$\begin{aligned} y_{k,s-d} &= \sum_{l=0}^{N-1} x_{l,s} H_{l,s-d} C_{l-k} + w_d^{(1)} \\ &= x_{k,s} H_{k,s-d} C_0 + \sum_{l=0, l \ne k}^{N-1} x_{l,s} H_{l,s-d} C_{l-k} + w_d^{(1)} \\ &= x_{k,s} H_{k,s-d} C_0 + I_{ICI(s-d)} + w_d^{(1)} \end{aligned} \tag{4}$$

Here x_l , $H_{k,s-r}$ and $H_{k,s-d}$ are the transmitted information symbol, channel coefficients of the source to relay (s-r) and the source to destination(s-d) links respectively. $I_{ICI(s-d)}$ and $I_{ICI(s-r)}$ are inter-carrier interference. C_L is defined as follows,

$$\begin{aligned} C_L &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j[(\frac{2\pi}{N})(L+\varepsilon)n + \phi(n)]} \\ &= \exp[j\{2\pi(L+\varepsilon)+\phi\}(1/2 - 1/2N)] \frac{\sin[\{2\pi(L+\varepsilon)+\phi\}/2]}{N.\sin[\{2\pi(L+\varepsilon)+\phi\}/2N]} \end{aligned} \tag{5}$$

Whereas, ε is the normalized frequency offset and given by $\Delta f T$. T is the subcarrier symbol period. In phase II, the source signal is received by the relay that is attenuated and it is needed to be amplified before transmission another time. Assuming that the channel characteristic is estimated perfectly, the desired received signal power is given by

$$E[|y_{k,s-r}|^2] = |x_{k,s}|^2 |H_{k,s-r}|^2 |C_0|^2 + P_{ICI(s-r)} + \sigma_r^2 \tag{6}$$

Where, w_r ~CN $(0, \sigma_r^2)$ and $w_d^{(1)}$ ~CN $(0, \sigma_d^2)$ are the AWGN at the relay and the destination respectively. σ_r^2 and σ_d^2 are the relay and destination noise variance. P_{ICI} is the inter-carrier interference power. If the channel gain $|H_{k,s-r}|^2$ is known at the relay, the relay uses amplification gain (β) [Hong et al., 2010],

$$\beta = \frac{1}{\sqrt{|x_{k,s}|^2 |H_{k,s-r}|^2 |C_0|^2 + P_{ICI(s-r)} + \sigma_r^2}} \tag{7}$$

The relay signal is found after multiplication of gain that is expressed by,

$$\begin{aligned} x_{k,r} &= \beta \times y_{k,s-r} \\ &= \sqrt{\frac{|x_{k,s}|^2}{|x_{k,s}|^2 |H_{k,s-r}|^2 |C_0|^2 + P_{ICI(s-r)} + \sigma_r^2}} H_{k,s-r,} C_0 \\ &+ \frac{I_{ICI(s-r)}}{\sqrt{|x_{k,s}|^2 |H_{k,s-r}|^2 |C_0|^2 + P_{ICI(s-r)} + \sigma_r^2}} \\ &+ \frac{1}{\sqrt{|x_{k,s}|^2 |H_{k,s-r}|^2 |C_0|^2 + P_{ICI(s-r)} + \sigma_r^2}} w_r \end{aligned} \tag{8}$$

The amplification gain (β) depends on the source to relay channel coefficient $H_{k,s-r}$. The relay forwards the amplified signal $x_{k,r}$ to the destination, where the received signal can be expressed as,

$$y_{k,r-d} = \sum_{l=0}^{N-1} x_{l,r} H_{l,r-d} C_{l-k} + w_d^{(2)}$$

$$y_{k,r-d} = x_{k,r} H_{k,r-d} C_o + \sum_{l=0, l\neq k}^{N-1} x_{l,r} H_{l,r-d} C_{l-k} + w_d^{(2)}$$

$$y_{k,r-d} = x_{k,r} H_{k,r-d} C_0 + I_{ICI(r-d)} + w_d^{(2)} \tag{9}$$

$$\begin{aligned} y_{k,r-d} &= \sqrt{\frac{|x_{k,s}|^2}{|x_{k,s}|^2 |H_{k,s-r}|^2 |C_0|^2 + P_{ICI(s-r)} + \sigma_r^2}} H_{k,s-r} H_{k,r-d} |C_0|^2 \\ &+ \frac{I_{ICI(s-r)}}{\sqrt{|x_{k,s}|^2 |H_{k,s-r}|^2 |C_0|^2 + P_{ICI(s-r)} + \sigma_r^2}} H_{k,r-d} C_0 \\ &+ \frac{1}{\sqrt{|x_{k,s}|^2 |H_{k,s-r}|^2 |C_0|^2 + P_{ICI(s-r)} + \sigma_r^2}} H_{k,r-d} C_0 w_r + I_{ICI(r-d)} + w_d^{(2)} \end{aligned}$$

The signals arriving at the destination can be utilized for detection with or without diversity combining. Now the both case are taken into consideration:

Condition 1: Without Diversity

In the case without diversity, only the signal $y_{k,r-d}$ can be utilized for detection at the destination. By (9), the received signal to noise interference ratio (SNIR) can be computed as,

$$\gamma_{SNIR\_WOD} = \frac{\gamma_{s-r}\gamma_{r-d} |C_0|^4}{1+\gamma_{s-r}(|C_0|^2 + \sum_{l=1}^{N-1}|C_{l-k}|^2) + \gamma_{r-d}(|C_0|^2 + \sum_{l=1}^{N-1}|C_{l-k}|^2) + \gamma_{s-r}\gamma_{r-d}(2|C_0|^2 \sum_{l=1}^{N-1}|C_{l-k}|^2 + \sum_{l=1}^{N-1}|C_{l-k}|^2 \sum_{l=1}^{N-1}|C_{l-k}|^2)} \tag{10}$$

In order to evaluate the statistical properties [Dwivedi et al., 2008], assuming average channel gain

$$E[|H_{k,s-d}|^2] = E[|H_{k,s-r}|^2] = E[|H_{k,r-d}|^2] = 1 \text{ and}$$

$$E[|x_{k,s}|^2] = E[|x_{l,s}|^2] = |x|^2 \tag{11}$$

Condition 2: With Diversity

In the case with diversity combination, the signals are received in Phases I and II. The signal can be combined in (4) and (9) at the destination using equal ratio combining (ERC) to obtain the output signal is expressed as,

$$\begin{aligned} y_{k,d} &= y_{k,s-d} + y_{k,r-d} \\ y_{k,d} &= x_{k,s} H_{k,s-d} C_0 + I_{ICI(s-d)} + w_d^{(1)} + y_{k,r-d} \end{aligned} \tag{12}$$

The effective SNIR at the output of the ERC is given by

$$\gamma_{SNIR\_ERC} = \frac{\gamma_{s-d} |C_0|^2}{1 + \sum_{l=1}^{N-1} |C_{l-k}|^2} + \gamma_{SNIR\_WOD} \tag{13}$$

Where, $\gamma_{s-d} = \dfrac{|x_{k,s}|^2}{\sigma_d^2}$ and $\gamma_{s-r} = \dfrac{|x_{k,s}|^2}{\sigma_r^2}$ are the signal to noise ratio. For a Nakagami-m fading channel, the probability density function (pdf) of γ is given by [M. S. Alouini, et al., 2000],

$$p_\gamma(\gamma_{SNIR\_WOD}) = \left(\frac{m}{\gamma_{SNIR\_WOD}}\right)^m \frac{\gamma_{SNIR\_WOD}^{m-1} \exp\left(-\dfrac{m\gamma_{SNIR\_WOD}}{\gamma_{SNIR\_WOD}}\right)}{\Gamma(m)}, \gamma \geq 0 \tag{14}$$

The bit error rate (BER) expression of two-dimensional Gray coding in coherent M-QAM over the AWGN channel [Quazi et al., 2011] can be expressed by,

$$p_b(\gamma_{SNIR\_WOD}) \cong 0.2 e^{\left(\frac{-3\gamma_{SNIR\_WOD}}{2(M-1)}\right)} \tag{15}$$

The average BER without diversity in Nakagami-m fading channel in (14) and (15) can be expressed as,

$$\begin{aligned} p_b(\gamma_{SNIR\_WOD}) &= \int_0^\infty p_b(\gamma_{SNIR\_WOD}) \times p_\gamma(\gamma_{SNIR\_WOD}) d\gamma_{SNIR\_WOD} \\ &= \int_0^\infty 0.2 e^{\left(\frac{-3\gamma_{SNIR\_WOD}}{2(M-1)}\right)} \times \left(\frac{m}{\gamma_{SNIR\_WOD}}\right) \frac{\gamma_{SNIR\_WOD}^{m-1} \exp\left(\dfrac{-m\gamma_{SNIR\_WOD}}{\gamma_{SNIR\_WOD}}\right)}{\Gamma(m)} d\gamma \\ &= 0.2 \left(\frac{m}{\gamma_{SNIR\_WOD} v}\right)^m \end{aligned} \tag{16}$$

Where, γ_{SNIR_WOD} = average SNIR and

$$v = \frac{3\gamma_{SNIR\_WOD} + 2m(M-1)}{2(M-1)\gamma_{SNIR\_WOD}}.$$

The average BER for ERC in Nakagami-m fading channel can be expressed as,

$$p_b(\gamma_{SNIR\_ERC}) = \int_0^\infty p_b(\gamma_{SNIR\_ERC}) \times p_\gamma(\gamma_{SNIR\_ERC}) d\gamma_{SNIR\_ERC}$$

$$= \int_0^{\infty} 0.2e^{\left(\frac{-3\gamma_{SNIR\_ERC}}{2(M-1)}\right)} \times \left(\frac{m}{\gamma_{SNIR\_ERC}}\right) \frac{\gamma_{SNIR\_ERC}^{m-1} \exp\left(\frac{-m\gamma_{SNIR\_ERC}}{\gamma_{SNIR\_ERC}}\right)}{\Gamma(m)} d\gamma$$

$$= 0.2 \left(\frac{m}{\gamma_{SNIR\_ERC}\,\nu}\right)^{m} \qquad (17)$$

Where, γ_{SNIR_ERC} = average SNIR and

$$\nu = \frac{3\gamma_{SNIR\_ERC} + 2m(M-1)}{2(M-1)\gamma_{SNIR\_ERC}}.$$

Results & Discussions

In this section, the average BER and SNIR have been evaluated from the derived equation integrating the effect of frequency offset and phase noise. The normalized frequency offset and phase noise are set to be 0.05 and 0.025 respectively. The M-ary, subcarrier, number of IFFT and FFT are taken to be 4, 64, 64 and 64 respectively.

Fig. 2 shows the performance of SNIR by varying different normalized frequency offset. From the figure 2, it is found that SNIR decreases for higher normalized frequency offset. At normalized frequency offset 0.15, the values of SNIR are approximately -4.160 and -1.750 for SNR=1 dB and 3 dB without diversity. However, the values of SNIR designed for ERC are approximately 1.750 and 3.80 for the same normalized frequency offset and SNR. For a typically assumed normalized frequency offset, ERC has better performance. The system provides a quite satisfactory performance for higher SNR.

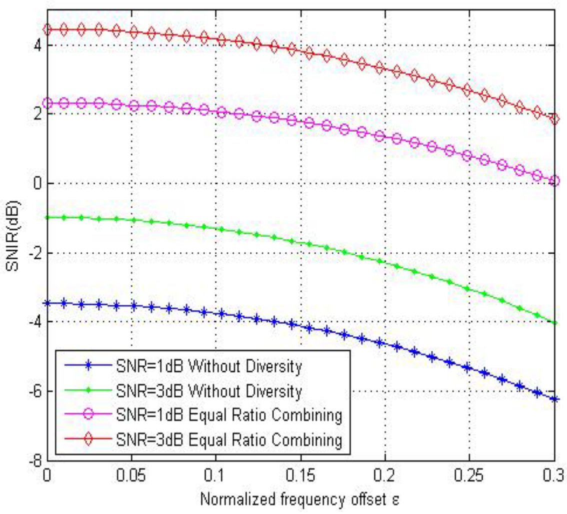

FIG. 2 PLOTS OF SNIR (DB) VS. NORMALIZED FREQUENCY OFFSET

Fig. 3 presents the BER performance without diversity technique. For a typically assumed SNR value of 12 dB, the average BER values are approximately 0.01868 and 0.04336 in case of *m=3* and *m=1*, the system performance is improved in *m=3* by approximately 3.65 dB as compared to *m=1* respectively. The system provides a quite satisfactory performance higher order Nakagami fading channel.

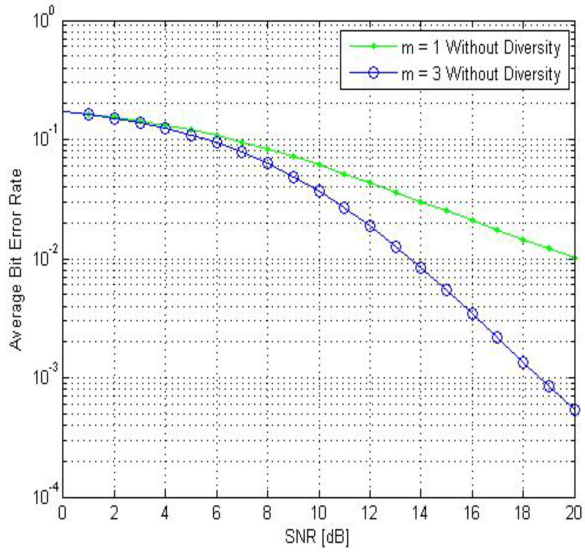

FIG. 3 PLOTS OF AVERAGE BER VS. SNIR WITHOUT DIVERSITY TECHNIQUE

Fig. 4 shows an illustration of BER of different Nakagami fading parameter under ERC technique. It is observed that the system with high order Nakagami fading parameter, ERC has better. For example, SNR value of 12 dB, *m=1* and *m=3* ERC in the system has the value of BER approximately 0.01656 and 0.00193 respectively. OFDM based AF relaying with ERC has the improvement approximately 9.33 dB.

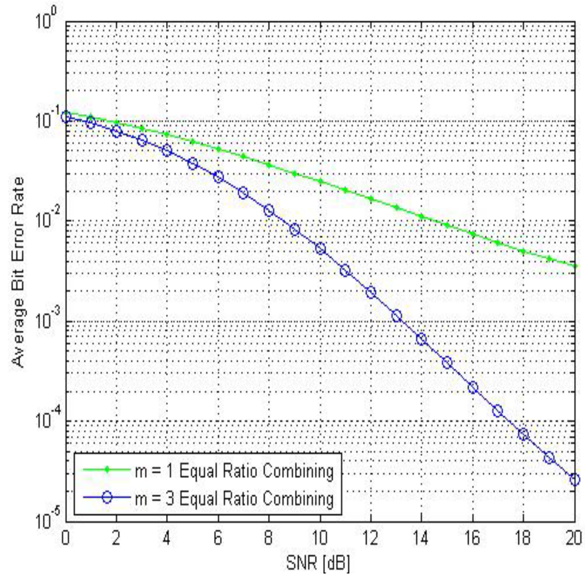

FIG. 4 PLOTS OF AVERAGE BER IN ERC TECHNIQUE

Fig. 5 shows the average BER of the OFDM based AF relaying system without diversity and ERC. At SNR value of 12 dB, the average BER values are 0.0046 and

0.0254 in case of *m*=2 ERC and *m*=2 without diversity schemes. It is observable that at high SNR value area, the system performance is comparatively better under deployment of the *m*=2 ERC technique.

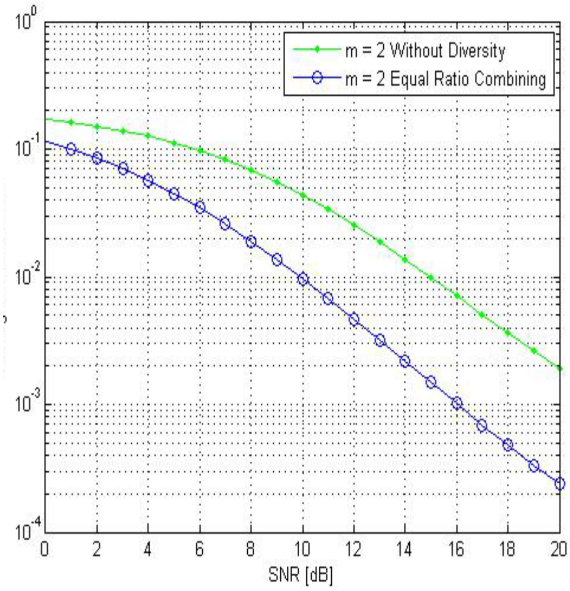

FIG. 5 PLOTS OF AVERAGE BER VS. SNIR WITHOUT DIVERSITY AND ERC

Conclusion

The performance of OFDM based AF relaying scheme without diversity and ERC in Nakagami fading channel has been analyzed. The derived average BER expression is efficient and offers a convenient way to evaluate the OFDM based AF relaying. Results showed that the performance of system degrades for large frequency offset and phase noise. The performance of system improves when SNR is high. The OFDM based AF relaying scheme with ERC integrating the effect of frequency offset and phase noise confirms the performance improvement.

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